EVIDENCE FOR IROSHNIKOV-KRAICHNAN-TYPE TURBULENCE IN THE SOLAR WIND
UPSTREAM OF INTERPLANETARY TRAVELING SHOCKS

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ABSTRACT

We analyze suprathermal ions and plasma wave spectra upstream of interplanetary shocks driven by coronal mass ejection events. In particular, we analyze the competition between two processes: (1) the upstream wave generation by suprathermal protons accelerated at the shock, and (2) the cascading of wave energy in the inertial range of solar wind turbulence. We derive the cascading timescale from the comparison of particle and turbulent wave spectra with theory and conclude that amplified solar wind turbulence upstream of interplanetary traveling shocks is better described by Iroshnikov-Kraichnan-type rather than Kolmogorov-type wave diffusion.

Subject headings: acceleration of particles — MHD — shock waves — solar wind — Sun: coronal mass ejections (CMEs) — turbulence

1. INTRODUCTION

In Bamert et al. (2004) particle spectra and wave power spectral densities in the plasma region upstream of the interplanetary shock driven by the Bastille Day coronal mass ejection (CME) of 2000 July 14, which passed SOHO on 2000 July 15, have been analyzed. For this event, the upstream wave activity in the frequency range 0.1 mHz to a few Hz and the spatial variation of suprathermal protons in the energy range 35 keV to a few MeV is fairly well described by the self-consistent quasi-linear theory of hydromagnetic wave generation and ion acceleration upstream of an interplanetary traveling shock by Lee (1983) and Gordon et al. (1999). However, no perfect match of data and theory could be obtained, in particular for proton energies below 60 keV.

Here the process of turbulent wave diffusion is included in the quasi-linear theory in order to test whether a more accurate match of data and model can be achieved. In addition, the studies are extended to other energetic (suprathermal) particle events, which are sufficiently strong to lead to the phenomenon of significant self-consistent upstream wave amplification near Earth’s orbit. The data associated with these events have been used to verify several conditions that must theoretically apply in order to give rise to upstream wave amplification above the turbulence level of the ambient solar wind:

1. The upstream suprathermal particle (proton) distribution needs to have a strong spatial gradient. This gradient builds up as a consequence of the balance between diffusion away from the shock and convection back into the shock.

2. The suprathermal particle flux must be sufficiently strong in order to cause an amplification of a given turbulence level of the ambient solar wind upstream of the shock.

3. The cascading timescale must be longer than the wave growth timescale.

Verification of the third condition in the wave and particle spectra allows us to distinguish between different heuristic turbulence models of the solar wind. In this work, the wave diffusion terms for Kraichnan-type and for Kolmogorov-type turbulence (Zhou & Matthaeus 1990) have been included in the quasi-linear theory (Lee 1983) in order to describe the process of nonlinear turbulent cascading.

2. OBSERVATIONS

In addition to the well-studied Bastille Day event in 2000 (Bamert et al. 2004), another large CME event has been analyzed, the event observed by SOHO LASCO and SOHO EIT on 2003 November 2 and 4. The latter event represents one of the two most active time periods in the declining phase of this solar cycle 23 (Woods et al. 2004). NOAA active region 10486 released two particularly strong flares, on 2003 November 2 17:30 UT (X8.3 at S14°W56°) and on 2003 November 4 19:29 UT (X28 at S19°W83°), the latter being the largest flare ever observed. Two fast CMEs have been associated with these flares (Lario et al. 2005), the first emerging with 2598 km s⁻¹ (2003 November 2) and the second with 2657 km s⁻¹ (2003 November 4). The interplanetary shocks driven by these CMEs arrived at SOHO on November 4 at 05:53 UT and on November 6 at 18:56 UT, respectively. Here we analyze the upstream region of the first of these two shocks for which upstream wave amplification similar to that during the Bastille Day 2000 event is observed.

We use data of the Highly Suprathermal Time-Of-Flight (HSTOF) spectrometer (Hovestadt et al. 1995; Bamert et al. 2002), which is part of the Charge, Element, and Isotope Analysis System (CELIAS) instrument package on board the Solar and Heliospheric Observatory (SOHO) (Domingo et al. 1995). The HSTOF sensor is a subsystem of a carbon-foil time-of-flight spectrometer which also contains the Suprathermal Time-Of-Flight (STOF) sensor. Both subsystems can measure mass and energy of suprathermal ions.

The magnetometer MAG (Smith et al. 1998) on board the Advanced Composition Explorer (ACE) measures the local interplanetary magnetic field (IMF) direction and magnitude and establishes both the large-scale structure and the fluctuation characteristics of the IMF at 1 AU upstream of Earth. These data allow the analysis of the wave activity in the solar wind plasma upstream of the strong interplanetary shocks.

Figure 1 shows an overview of spacecraft data associated with the Bastille Day CME. The turbulent spectrum (bottom
panel) upstream of the interplanetary shock is characterized by two wavenumbers, $k_{\min}$ and $k_{\max}$. Below $k_{\min}$ the power spectral density follows a power law representing the turbulent cascade of the power spectral density of the ambient solar wind. The spectral index of this power law is $-1.65 \pm 0.1$. Between $k_{\min} \approx 0.5k_{1\ MeV}$ ($k_{1\ MeV}$ being the wavenumber of parallel Alfvén waves that are in gyroresonance with 1 MeV protons propagating along the ambient magnetic field) and $k_{\max} \approx 2k_{1\ MeV}$, the power spectral density increases above the level of turbulent power in the ambient solar wind. Above $k_{\max}$ the power spectral density again resembles the spectrum of a turbulent cascade, although at a higher level. The spectral index of the power law above $k_{\max}$ is $-1.5 \pm 0.25$.

The Halloween 2003 event (2003 November 4; Fig. 2) shows a similar behavior. In the wavenumber range $0.2k_{1\ MeV} < k < 2k_{1\ MeV}$ the power spectral density rises above the level of the ambient solar wind turbulence, while above $k_{\max} \approx 2k_{1\ MeV}$ the spectrum again follows a power law. For this event, the ambient solar wind turbulence has a power spectral index of $-1.65 \pm 0.15$, while the amplified waves have a power spectral index of $-1.5 \pm 0.25$ at wavenumbers larger than $k_{\max}$.

3. DISCUSSION

Isotropic hydrodynamic turbulence is usually described by Kolmogorov’s (1941) self-similarity theory, which predicts a power spectral index of $-5/3$ in the turbulent cascades. Magnetohydrodynamic turbulence has first been described by Iroshnikov (1964) and Kraichnan (1965), who postulate a spectral index of $-3/2$. In the solar wind, both types of scaling have been found for different time intervals (Veltri 1999; Bershadskii 2002). Recent work by Chapman & Hnat (2007) based on the analysis of structure functions suggests that both types of turbulence coexist in the anisotropic solar wind.

Here we take a different approach by directly determining the cascading timescale which is significantly different for Kolmogorov and Iroshnikov-Kraichnan turbulence. The cascading timescale is derived using the observed parameter $k_{\min}$ indicating the wavenumber where the timescale of upstream wave growth equals the cascading timescale.

We briefly outline our approach as follows. The quasi-linear theory of upstream wave generation (Lee 1983) assumes an anisotropy proportional to the pitch angle of the protons in the distribution function of the energetic protons entering the upstream plasma. These protons amplify the antisunward propagating Alfvén waves. Their relative power spectral density $P(k, z) = dB^2(k)/B_0^2$ at distance $z$ upstream from the shock, and the omnidirectional phase-space density of the protons $F_p(k, z)$ are related by a growth factor $\gamma_p(k)$:

$$P(k', z) = \gamma_p(k')F_p(k', z) + P(k', \infty),$$

$$P(k', \infty) = P_0k'^{5/3}, \quad k' = k/k_{1\ MeV}. \quad (1)$$

The function $F_p$ solves the above equation and the transport equation simultaneously:

$$F_p(k, z) = \frac{F_p(k, 0)}{[1 + g(k)] \exp[h(k)z] - g(k)};$$

$$g(k) = \frac{\gamma_p(k)F_p(k, 0)}{P(k, \infty)}, \quad \gamma_p(k) = \frac{3\pi^2\Omega_p^3}{\beta_{sh}(\beta_{sh} - 2)k^2N_pV_pV_A},$$

$$h(k) = \frac{V_{sw}P(k, \infty)}{\kappa_p(k)}, \quad \kappa_p(k) = \frac{\Omega_p}{8\pi k^3}. \quad (2)$$

with $V_{sw}$ the upstream solar wind speed. At the shock ($z = 0$), the energetic protons usually have a power-law distribution $F_p(k', 0) = F_0k'^{3/2}$. For the parameters of the Bastille Day event (Bamert et al. 2004), we have derived theoretical functions fitted to observations:

$$g(k') = g_0k'^{1/3}h_{0}^{-1/3}, \quad g_0 = 7.5,$$

$$h(k') = h_0k'^{4/3} \text{AU}^{-1}, \quad h_0 = 1.65. \quad (3)$$
Another way of writing the evolution of turbulent power is

\[
\phi(k', z) = P(k', z) / P(k', \infty) = g(k') Z(k', z) + 1,
\]

\[
Z(k', z) = \left(1 + g(k') \right) \exp \left[h(k') z - g(k')^{-1}\right]
\]

\[
\approx 1 - h(k') g(k') z.
\]

The timescale for wave growth is—normalized to the parameters for the Bastille Day event (subscript B; Bamert et al. 2004; Table 1) for convenient comparison with data—

\[
\tau_g(k') = \frac{Z(k', z)}{V_w} \frac{1}{dZ(k', z)/dz} \approx 2 \times 10^4 \text{s} \, k'(1 - \beta_{al}) A_{B,B},
\]

\[
A_{B,B} = \left(\frac{N_p}{N_{p,B}}\right) \left(\frac{V_{sw,B}}{V_w}\right) \left(\frac{V_a}{V_{sw,B}}\right) \left(\frac{F_{0,B}}{F_0}\right) \left[\beta_{sh,B} (\beta_{sh,B} - 2)\right].
\]

Two timescales govern the upstream wave spectra:

1. The nonlinear eddy turnover time \(\tau_{nl}\) corresponding to an energy transfer time \(\tau_E = \tau_{nl} (\tau_{al}^2 / \tau_{nl}^2)\) with \(\tau_{nl} = (k h)^{-1}\) the Alfvén timescale.

2. The growth timescale \(\tau_g(k')\) of Alfvén waves due to anisotropic energetic protons with phase-space density \(F_p(k', z)\).

The waves are amplified to amplitudes above the turbulence level of the ambient solar wind once the growth timescale \(\tau_g\) becomes shorter than the energy transfer time \(\tau_E\). This occurs at sufficiently large wavenumbers \(k'\):

\[
\tau_{nl}(k') < \tau_E(k') \approx \tau_{nl} \left(\frac{\tau_{al}^2}{\tau_{nl}^2}\right) (k')
\]

\[
\approx 200 \text{s} \, k' \left(\frac{V_{sw}}{V_a}\right) \left(\frac{\Omega_{p,B}}{\Omega_p}\right) \left(\frac{\tau_{al}^2}{\tau_{nl}^2}\right) |k'|.
\]

The condition for wave growth finally reads \(k' > k'_{nl}\) with

\[
(k'_{nl})^{(1 - \beta_{al})} < 100 \frac{\tau_{al}^2}{\tau_{nl}^2},
\]

\[
A_{B,B} = \left(\frac{N_p}{N_{p,B}}\right) \left(\frac{V_{sw,B}}{V_w}\right) \left(\frac{V_a}{V_{sw,B}}\right) \left(\frac{F_{0,B}}{F_0}\right) \left[\beta_{sh,B} (\beta_{sh,B} - 2)\right].
\]

Only for the Bastille Day and the Halloween event are the parameters \(k_{min}\) and \(k_{max}\) identifiable in the data. For the two other events the spatial gradient of the suprathermal protons may be reduced due to suprathermal particles already present in the upstream region from the preceding flares, which, therefore, results in a too low growth rate for upstream waves.

Evaluation of the data for the Bastille Day event and the Halloween event (Table 2) yields eddy turnover times which are considerably larger than the Alfvén time. This suggests that the ambient solar wind turbulence is rather weak far upstream of the Bastille Day event and the Halloween event. For strong coupling, one expects \(\tau_{al} \approx \tau_{nl}\) while \(\tau_{al} \gg \tau_{nl}\) characterizes the case of weak coupling of the turbulent modes. A somewhat more quantitative model can be given by putting the wave diffusion term into the wave transport equation. The cascading term in the case of isotropic Kolmogorov-type hydrodynamic turbulence is (Zhou & Matthaeus 1990)

\[
\frac{\partial}{\partial k} \left[ D_{kk}^+ \frac{\partial P[k, z]}{\partial k}\right], \quad D_{kk}^+ = \frac{1}{\tau_{al}} \sqrt{P[k, z] \frac{k^5}{2}}.
\]

**Table 1**

<table>
<thead>
<tr>
<th>Observation</th>
<th>(B_0) (nT)</th>
<th>(N_p) ((\text{cm}^{-3}))</th>
<th>(V_{sw}) ((\text{km s}^{-1}))</th>
<th>(V_a) ((\text{km s}^{-1}))</th>
<th>(\beta)</th>
<th>(\beta_{al})</th>
<th>(\theta_{al})</th>
<th>(F_0) ((10^{-4} \text{ s}^{-1} \text{km}^{-3} \text{sr}))</th>
<th>(P) ((10^5 \text{ m}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000 Jul 14–15 ......</td>
<td>10 ± 0.7</td>
<td>6.2 ± 0.3</td>
<td>561 ± 18</td>
<td>88 ± 5</td>
<td>1 ± 0.1</td>
<td>4.9 ± 0.2</td>
<td>144 ± 7</td>
<td>4.7 ± 0.3</td>
<td>4.7 ± 0.3</td>
</tr>
<tr>
<td>2003 Nov 2–4 ......</td>
<td>8 ± 0.5</td>
<td>6.7 ± 0.9</td>
<td>491 ± 12</td>
<td>68 ± 7</td>
<td>1 ± 0.1</td>
<td>4.5 ± 0.2</td>
<td>75 ± 10</td>
<td>2.7 ± 0.2</td>
<td>3.9 ± 0.2</td>
</tr>
</tbody>
</table>

\* \(\theta_{al}\) is calculated using the model described in Gonzales-Esparza & Balogh (2001); ACE MAG data are taken from the Web site “MAG 16-second Averaged Interplanetary Magnetic Field Data” (http://www.srl.caltech.edu/ACE/ASC/level1/level1DATA_MAG.html).

**Table 2**

<table>
<thead>
<tr>
<th>Observation</th>
<th>(F_{min})</th>
<th>(k_{max})</th>
<th>(A_B)</th>
<th>(A_{B,B})</th>
<th>(\tau_{nl}/H_a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000 Jul 14–15 ......</td>
<td>0.5 ± 0.05</td>
<td>2 ± 0.5  2*</td>
<td>1 ± 0.08</td>
<td>1 ± 0.08</td>
<td>14.0 ± 1.7</td>
</tr>
<tr>
<td>2003 Nov 2–4 ......</td>
<td>0.3 ± 0.1</td>
<td>2 ± 0.5  2*</td>
<td>0.98 ± 0.12</td>
<td>1.44 ± 0.23</td>
<td>13.5 ± 1.6</td>
</tr>
</tbody>
</table>

\* At 0.015 AU upstream of the shock.

\* At 0.012 AU upstream of the shock.
Plugging the solution \( \phi_0(k', z) = g(k') Z(k', z) \) into the wave diffusion term yields, as a first-order approximation,

\[
\frac{d\phi}{dz}(k', z) = g(k') \frac{dZ}{dz}(k', z) - \frac{\alpha_w k^{5/3}}{\nu_a \tau_{\alpha,1}} \frac{\partial}{\partial k'} \left[ \sqrt{g(k')} Z(k', z) \frac{\partial}{\partial k'} \left( \frac{g(k')}{k^{5/3}} Z(k', z) \right) \right],
\]

with

\[
\alpha_w = \sqrt{2P(k' = 1, \infty)} k_{1\text{MeV}},
\]

\[
\alpha_{w,B} \approx 0.015, \quad \tau_{\alpha,1} = \tau_a(k' = 1).
\] (10)

An approximate solution is

\[
\phi(k', z) \approx g(k') \left[ 1 - \gamma_c k^{5/3} \right] Z(k', z) + 1,
\]

\[
\gamma_c = \frac{\sqrt{2P_{k_{1\text{MeV}}}} (6 - \beta_{sh})(3\beta_{sh} - 13)}{\sqrt{2g(k)\nu_a \tau_{\alpha,1}}},
\]

\[
\gamma_{c,B} \approx 4, \quad \gamma_{c,H} \approx 0.6.
\] (11)

For isotropic magnetohydrodynamic Iroshnikov-Kraichnan turbulence, involving the Alfvén timescale in the description of the cascading process (Zhou & Matthaeus 1990), the wave diffusion parameter and the resulting solution of the wave equation is

\[
D_{1\ell}^{*} = \frac{1}{\tau_{\alpha}} k^{13/3} P(k, z) \Rightarrow \phi(k', z)
\]

\[
\approx g(k') \left[ 1 - \delta_a k^{5/3} \right] Z(k', z) + 1,
\]

\[
\delta_a = \frac{P_{k_{1\text{MeV}}}(6 - \beta_{sh})(6\beta_{sh} - 14)}{3h_{0} \nu_a \tau_{\alpha,1}},
\]

\[
\delta_{a,B} \approx 0.5, \quad \delta_{a,H} \approx 0.3.
\] (12)

The Kolmogorov theory predicts the onset of wave growth at \( k_{\text{min,B}} \approx 4 \) for the Bastille event and at \( k_{\text{min,H}} \approx 0.7 \) for the Halloween event (denoted H), while the Iroshnikov-Kraichnan theory predicts \( k_{\text{min,B}} \approx 0.5 \) and \( k_{\text{min,H}} \approx 0.3 \). Apparently, the Iroshnikov-Kraichnan-type wave diffusion term describes the data better. This is not surprising as the solar wind represents a magnetohydrodynamic system, and mainly Alfvénic fluctuations are amplified upstream of interplanetary traveling shocks. We presumably observe the competition of Alfvén wave growth described by the quasi-linear theory (Lee 1983) with cascading of these Alfvén waves described by the theory of Iroshnikov (1964) and Kraichnan (1965) rather than by the theory of Kolmogorov (1941). More refined comparisons to theory are beyond the scope of this article. For instance, no wave diffusion term for anisotropic turbulence has been derived so far.

We finally conclude from the determination of the cascading timescale that the heuristic model of magnetohydrodynamic Iroshnikov-Kraichnan-type wave diffusion describes amplified solar wind turbulence upstream of interplanetary traveling shocks significantly better than the heuristic model of hydrodynamic isotropic Kolmogorov-type wave diffusion.

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